Elliptic Curves @ UConn
Titles and Abstracts

All talks will be at the Biology/Physics Building Room 131 (BPB 131), and the coffee breaks will be outside of BPB 131. A campus map pointing to BPB can be found here (Google labels the building as the Dept of Physics).

- **Keith Conrad** (University of Connecticut), 10 - 10:50.

  Title: *Using elliptic curves.*

  Abstract: While elliptic curves are studied for their own sake as central objects in number theory, they also appear in the solution to problems that look at first to be about something else entirely. We will discuss a few such "unexpected" applications, where algorithms, theorems, or conjectures related to elliptic curves answer questions that are not directly about elliptic curves.

- **Filip Najman** (University of Zagreb), 11:10 - 12.

  Title: *Mordell-Weil groups of elliptic curves over number fields*

  Abstract: The Mordell-Weil group of an elliptic curve over a number field is isomorphic to $T \oplus \mathbb{Z}^r$, where T is the torsion subgroup and r is the rank of the elliptic curve. We will survey known results about the torsion and rank of elliptic curves over number fields, and also how these two values interact. We will show that certain properties (concerning the rank and the field of definition) are forced onto an elliptic curve with a given torsion group over a number field of degree d and that these results arise from geometric properties of modular curves.

- **David E. Rohrlich** (Boston University), 2 - 2:50.

  Title: *Birch and Swinnerton-Dyer with twist*

  Abstract: Let F be a number field, E be an elliptic curve over F, and τ an irreducible Artin representation of F. The natural action of $\mathrm{Gal}(\overline{F}/F)$ on $E(\overline{F})$ affords a linear representation of $\mathrm{Gal}(\overline{F}/F)$ on $\mathbb{C} \otimes_{\mathbb{Z}} E(\overline{F})$, and the Conjecture of Birch and Swinnerton-Dyer with Twist asserts that

  $$\mathrm{ord}_{s=1} L(s, E, \tau) = m_\tau(E), \tag{1}$$

  where $m_\tau(E)$ is the multiplicity of τ in $\mathbb{C} \otimes_{\mathbb{Z}} E(\overline{F})$. We recover the usual Birch and Swinnerton-Dyer conjecture by taking τ to be the trivial character of $\mathrm{Gal}(\overline{F}/F)$. Now let $W(E, \tau)$ be the root number associated to E and τ. If τ is self-dual, then it follows from (1) that

  $$W(E, \tau) = (-1)^{m_\tau(E)}. \tag{2}$$

  We shall illustrate (2) with some examples.

- **Noam Elkies** (Harvard University), 3:10 - 4.

  Title: *Elliptic curves and surfaces, and their Mordell-Weil groups*

  Abstract: An elliptic curve E_t/k depending on a parameter t is simultaneously an elliptic curve $\mathcal{E}$ over the function field $k(t)$ and an elliptic surface S_E over k (that is, an algebraic surface S over k together with a rational function t whose generic fiber is an elliptic curve). These two points of view interact fruitfully; for example, a rational point on $\mathcal{E}$ is a section of the map t on S, and by using intersection theory on S we can recover the structure of the Mordell-Weil group $\mathcal{E}(k(t))$ from the Néron-Severi lattice of S. We explain this connection, and describe some applications including the construction of curves of record rank for $k = \mathbf{Q}$.

- **Rob Benedetto** (Amherst College), 4:20 - 5:10.

  Title: *Elliptic curves over p-adic fields*

  Abstract: Let K be a complete non-archimedean field, such as the p-adic field $\mathbb{Q}_p$, and let E be an elliptic curve defined over K. The non-archimedean metric on K makes it possible to apply analytic methods to study the set $E(K)$ of K-rational points on E. In this talk, we will touch on some such methods, including Néron models, Tate curves, and, if time permits, Berkovich spaces. Our emphasis will be on examples and ideas rather than on a rigorous presentation of the theory.