

A QUICK INTRO TO A.G. (CONTINUES)

PREVIOUSLY...

- Affine and projective space
- Affine and projective algebraic sets and their ideals.

- Varieties (when ideal is prime)

- Coordinate ring and function field of a variety

- Dimension (transcendence degree of $\bar{K}(V)/\bar{K}$)

- Curve: proj. variety of dimension 1.

- $C: f(x,y,z)=0$ is singular at $P \in C \iff \frac{\partial f}{\partial x}|_P = \frac{\partial f}{\partial y}|_P = \frac{\partial f}{\partial z}|_P = 0$.

- Local ring of V at P

$$\bar{K}[V]_P = \left\{ F \in \bar{K}(V) : F = \frac{f}{g}, f, g \in \bar{K}[V], g(P) \neq 0 \right\}$$

" F is regular at P "

$$V = \mathbb{P}^1$$

$$\mathbb{Q}[x]$$

$$\bar{K}(V) = \text{fraction field of } \bar{K}[V] \quad \mathbb{Q}(x)$$

$$\bar{K}[V] = \frac{\bar{K}[x]}{I(V)}$$

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- Rational map: $\phi: V_1 \longrightarrow V_2$
 $P \longmapsto [f_0(P), \dots, f_n(P)] \in V_2$

where $f_i \in \overline{k}(V_1)$.

- A rational map ϕ is regular at P if $\exists g \in \overline{k}(V_1)$ s.t.
 - (i) each $g \cdot f_i$ is regular at P
 - (ii) for some i we have $(g \cdot f_i)(P) \neq 0$.

ex $\phi: \{zy^2 = x^3 + z^3\} \longrightarrow \mathbb{P}^1$ is regular at $P = [0, 1, 0]$
 $[x, y, z] \longmapsto [x, z]$

$$\begin{aligned} \text{Take } g = x^2. \text{ Then } [x, z] &= [x^3, zx^2] \\ &= [zy^2 - z^3, zx^2] \\ &= [y^2 - z^2, x^2] \end{aligned}$$

- If ϕ is regular at all $P \in V_1$,
we call it a morphism.

evaluated at P
is $[1, 0]$ ✓

TODAY: ALGEBRAIC CURVES.

Let C be a curve (proj. variety of dimension 1) and let P be a smooth point.
 $M_P = \{f \in \bar{K}[C]_P : f(P) = 0\}$

DEF. • $\text{ord}_P : \bar{K}[C]_P \longrightarrow \mathbb{Z}^{\geq 0} \cup \{\infty\}$
 $\text{ord}_P(f) = \sup \{d \in \mathbb{Z} : f \in M_P^d\}$
extend by $\text{ord}_P(f/g) = \text{ord}_P(f) - \text{ord}_P(g)$ to $\bar{K}(C)$.

$\bar{K}[C]_P$ is a DVR.

• A uniformizer at P is a fn. $t \in \bar{K}(C)$ s.t. $\text{ord}_P(t) = 1$.

PROP. Let $f \in \bar{K}(C)$, then there are finitely many zeros and poles of f .
If f has NO poles, then $f \in \bar{K}$. (C is smooth)

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$$y^2 = x^3 + Ax + B =$$

