

PREVIOUSLY... Let C be a curve.

- $\text{Div}(C) = \left\{ D = \sum_{P \in C} n_P \cdot (P) : n_P \in \mathbb{Z}, n_P = 0 \text{ for almost all } P \in C \right\}$

- $\deg D = \sum_{P \in C} n_P$

- $\text{Div}^0(C) = \{ D \in \text{Div}(C) : \deg D = 0 \}$

- $f \in \overline{K}(C)^*$, $\text{div}(f) = \sum_{P \in C} \alpha d_P(f) \cdot (P)$

- $\text{Princ}(C) = \{ D : D = \text{div}(f), f \in \overline{K}(C)^* \}$

- $\text{Pic}(C) = \text{Div}(C) / \text{Princ}(C)$, $\text{Pic}^0(C) = \text{Div}^0(C) / \text{Princ}(C)$

- $\Omega_C = \{ dx : x \in \overline{K}(C) \} / d(x+y) = dx+dy, dx \cdot y = x \cdot dy + y \cdot dx, da = 0 \text{ if } a \in \overline{K}$.

↳ Ω_C is a 1-dim'l space over $\overline{K}(C)$

↳ if t is a unif. at a smooth pt $P \in C \Rightarrow dt$ is a basis of $\Omega_C / \overline{K}(C)$.

- $\text{div}(\omega) = \sum_{P \in C} \alpha d_P(\omega) (P) = \sum_{P \in C} \alpha d_P \left(\frac{\omega}{dt_P} \right) \cdot (P)$

→ $\text{div}(f) = 0 \Leftrightarrow f \in \overline{K}^*$

→ $\deg(\text{div}(f)) = 0$

↳ ex $\text{div}(x - e_i) = 2 \cdot (P_i) - 2 \cdot (O)$

$\text{div}(y) = (P_1) + (P_2) + (P_3) - 3 \cdot (O)$

PREVIOUSLY...

- ex $F_2 \mathbb{P}^1$, $\Omega_{\mathbb{P}^1} = \langle dx \rangle$, $\text{div}(dx) = -2 \cdot ([1, 0])$
- ex $F_2 E: y^2 = (x-e_1)(x-e_2)(x-e_3)$, $\Omega_E = \langle \frac{dx}{y} \rangle$, $\text{div}\left(\frac{dx}{y}\right) = 0$.

we say $\frac{dx}{y}$ is a non-vanishing, holom. differential.

- K_C , the canonical divisor class on C is the class of $\text{div}(w) \in \text{Pic}(C)$ for any non-zero differential $w \in \Omega_C$.

→ on \mathbb{P}^1 , $K_{\mathbb{P}^1} = [-2 \cdot ([1, 0])]$.

→ on E , $K_E = [0]$.

... TODAY ...

RIEMANN - ROCH!

TODAY... RIEMANN - ROCH!



Bernhard



