

ELLIPTIC CURVES OVER LOCAL FIELDS

$$K, R, \mathcal{M}, \pi, k = R/\mathcal{M}.$$

$$E'/K \rightsquigarrow \text{minimal model } E/K \text{ with } \begin{cases} a_i \in R, \\ \text{minimal } \nu(\Delta). \end{cases}$$

$$\begin{array}{c} \downarrow \text{reduction} \\ \text{mod } \mathcal{M} \\ \tilde{E}/k \end{array}$$

$$\begin{cases} \tilde{E}_{ns}(k) & \text{non-sing. points on } \tilde{E} \\ E_0(k) = \{P \in E(k) : \tilde{P} \in \tilde{E}_{ns}(k)\} \\ E_1(k) = \{P \in E(k) : \tilde{P} = \tilde{O}\} \end{cases}$$

THM. • There is an exact sequence:

$$0 \rightarrow E_1(k) \rightarrow E_0(k) \xrightarrow{\sim} \tilde{E}_{ns}(k) \rightarrow 0$$

$$\begin{array}{c} \cong \\ \hat{E}(\mathcal{M}) \end{array}$$

$$\bullet E_1(k) \cong \hat{E}$$

