

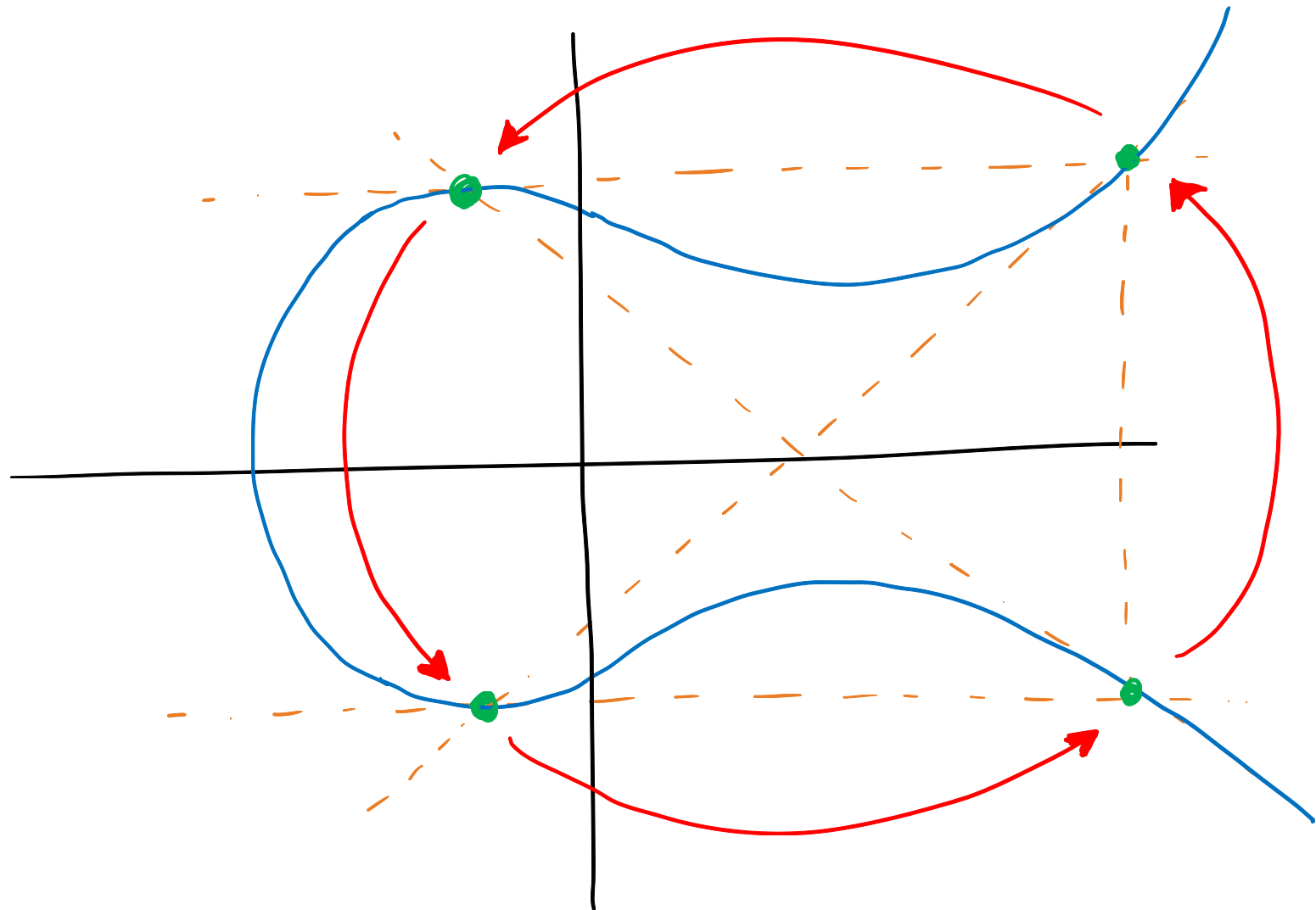
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“Towards a classification of
adelic Galois representations
attached to elliptic curves over \mathbb{Q} ”

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E/\mathbb{Q} an elliptic curve.

$$\rho_E : \text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \text{GL}(2, \hat{\mathbb{Z}})$$



Let E/\mathbb{Q} be an elliptic curve.

Thm (Mordell-Weil) $E(\mathbb{Q})$ is a finitely generated abelian group.

- $E(\overline{\mathbb{Q}})$ is **NOT** finitely generated
but its torsion subgroup is **well-understood!**

$$E(\overline{\mathbb{Q}})_{\text{tors}} = \bigcup_{n \geq 2} E[n]$$

$$\text{and } E[n] = \underbrace{E(\overline{\mathbb{Q}})[n]}_{n\text{-torsion subgroup}} \cong \mathbb{Z}/n\mathbb{Z} \oplus \mathbb{Z}/n\mathbb{Z}.$$

$E[n] \cong \mathbb{Z}/n\mathbb{Z} \oplus \mathbb{Z}/n\mathbb{Z} \hookrightarrow \text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \dots$ fixed points?
 $G_{\mathbb{Q}}$

• Over \mathbb{Q} , however...

Thm (Mazur) $E(\mathbb{Q})_{\text{tors}} \cong \begin{cases} \mathbb{Z}/N\mathbb{Z} & N=1, 2, \dots, 10, \text{ or } 12, \text{ or} \\ \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2M\mathbb{Z} & M=1, 2, 3, \text{ or } 4. \end{cases}$
 (and only many j's for each possibility!)

• How about (cyclic) $G_{\mathbb{Q}}$ -invariant subgroups? (not just pointwise-fixed)

Thm (Fricke, Kenku, Klein, Kubert, Ligozat, Mazur, Ogg, ...)

If $\langle P \rangle \subseteq E(\bar{\mathbb{Q}})$ is finite, $G_{\mathbb{Q}}$ -invariant, then

$\langle P \rangle \cong \mathbb{Z}/N\mathbb{Z} \iff \begin{cases} N=1, \dots, 10, \text{ or } 12, 13, 16, 18, 25 & (\text{only many } j\text{'s for each possibility}) \\ N=11, 14, 15, 17, 19, 21, 27, 37, 43, 67, \text{ or } 163 & (\text{only } < \infty \text{ } j\text{'s}) \end{cases}$
 $j = -2^{15} \cdot 3 \cdot 5^3$

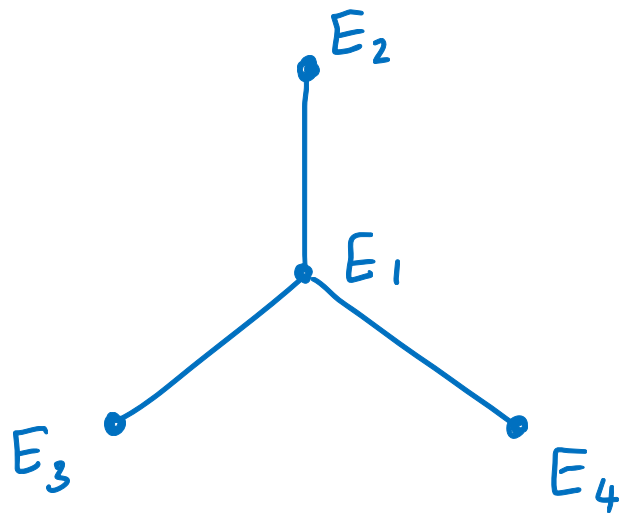
NOTE: $\{ \langle P \rangle \curvearrowright G_{\mathbb{Q}} \} \leftrightarrow \left\{ \begin{array}{l} \text{isogenies} \\ E \rightarrow E' / \mathbb{Q} \\ \text{w/ cyclic kernel} \end{array} \right\}$

• Combine previous two results ... $E / \mathbb{Q} \longrightarrow E' / \mathbb{Q}$ an isogeny.

Q. What are the possible combinations

$(E(\mathbb{Q})_{\text{tors}}, E'(\mathbb{Q})_{\text{tors}})$?

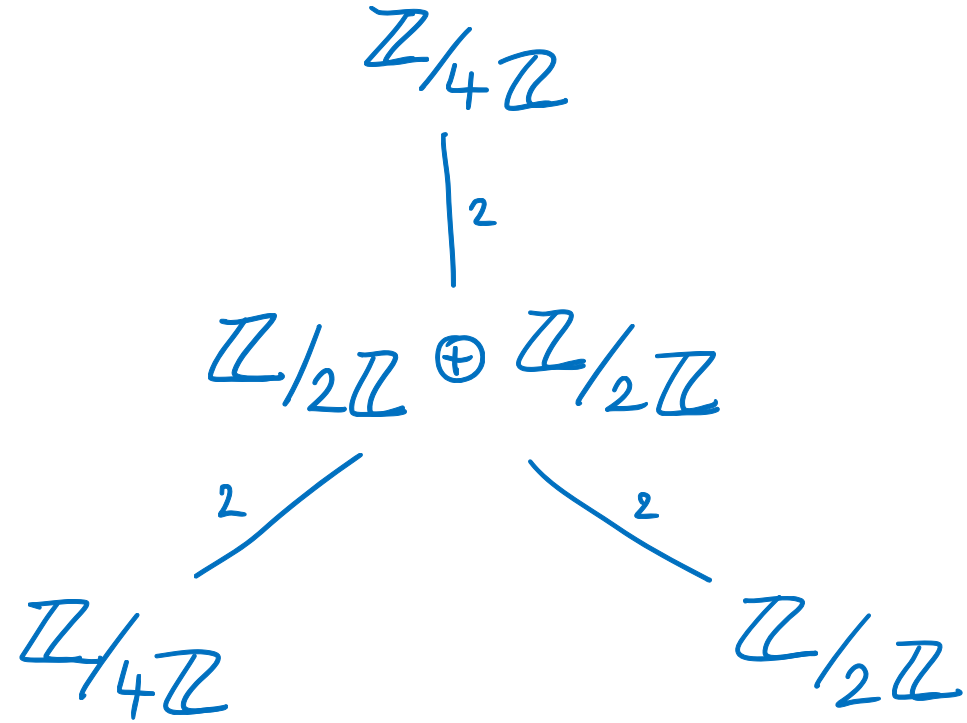
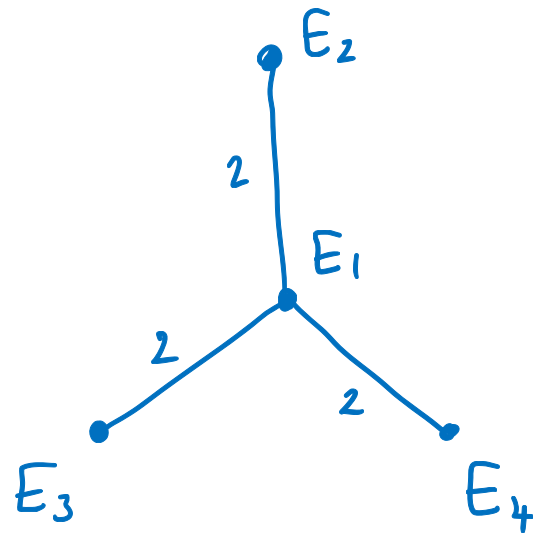
More generally...



What are the possibilities
for

$(E_i(\mathbb{Q})_{\text{tors}})_{i=1}^4$?

Example $E = E_1 : y^2 + xy + y = x^3 - x^2 - 6x - 4$
 (17.a2)



Thm (Chiloyan, L-R.)

There are 52 iso. types of isoge

