

# THE WEAK! MORDELL-WEIL THEOREM

Thm Let  $K$  be a number field, and  $m \geq 2$ , and  $E/K$  an ell. curve.

Then,  $E(K)/mE(K)$  is finite.

Lemma If  $K'/K$  is finite Galois, and  $E(K')/mE(K')$  is finite, then  $E(K)/mE(K)$  is finite.

**UPSHOT:** ASSUME  $E[m] \subseteq E(K)$ .

DEF. The Kummer Pairing  $\kappa : E(K) \times \text{Gal}(\bar{K}/K) \rightarrow E[m]$   
 $(P, \sigma) \mapsto Q^\sigma$























