

THE DAY SHE PROVED IT

ÁLVARO LOZANO-ROBLEDO



Claudia Suárez woke up and immediately knew she had solved it.

It had been one of those nights when her subconscious mind kept working on mathematics all night long, while her conscious self tried to rest in vain. She had been tossing, turning, tossing and turning, while her neurons, taking no prisoners, kept inexorably marching towards a proof of the conjecture she was obsessed with. A few times, she awoke briefly, in a daze, and fell back into a light slumber only to wake up again after what seemed just a few minutes of poor quality sleep. During the rare occasions when she actually reached a deeper part of a sleep cycle, her dreams confounded plausible reality with abstract concepts from her research in disconcerting ways that triggered spikes in her anxiety levels. In what was a recurring dream, a surgeon had to remove a cohomological theory from her brain to save her life and, with the first incision, she would abruptly wake up still feeling the pain of sharp needles digging into her scalp. Groggy and confused once again after the disturbing nightmare, she had a few sips of water in an effort to clear her head. When she tried to go back to sleep, the tossing and turning resumed, while thoughts raced through her head, running over all the possible ways in which she could put together the last pieces of her mathematical puzzle.

Now that she was fully awake, though, she knew she had gotten it. Claudia knew how to do it. The solution was natural, elegant, and she had a strong hunch that it would work. The idea was in fact so natural that she kicked herself for not coming up with it any earlier. There was barely any light outside, but she could not wait any longer to map out the entire argument on her notepad, and then to type it up in all its glory in order to make sure all the details checked out.

Richard was still fast asleep in bed beside her – it was just a few minutes before 6am – so she got up as quietly as she could and tip-toed out of the bedroom and into the spare room that served as an office in their apartment. When she turned on a light, she was surprised to find a full cup of coffee on the desk, next to a hand-written note from Rick – “You can do it!” – decorated with a goofy-looking smiley face. The coffee was cold, just the way she liked it on

an early summer morning, but clearly brewed not long ago... Did he brew coffee for her before going to bed? She could not recall, but it was a sweet surprise nonetheless, and she made a mental note to give him a big hug as soon as he woke up.

Claudia hesitated before turning on her laptop. If she was right, if this new argument worked, the paper she had worked on so hard for so long would finally be complete. This article would be a huge deal, as it would settle one of the longest standing conjectures in her field. Of course, it would also be game-changing for her career. She had done quite well for herself until now (she was a postdoc at a prestigious research institution), but this paper would catapult her name into the annals of mathematics, and not just into the journal of that name.

Prior to this point, she had spent many months, years in fact, learning about the conjecture, reading books and papers on the subject, going to conferences related to it, computing examples, trying different approaches, and learning new methods. A few months ago, she had a breakthrough, finding a way to prove a special case. Even that would have made quite a splash, but she soon realized that, perhaps, the same method could be pushed to prove the entire conjecture. So, she kept her discovery a secret, and months of hard work ensued, which seemed to get her closer, inch by inch, to a full proof. During the last week or so, she was so close to a complete argument that she could taste it, but the last few details had eluded her.

Until now.

“One step at a time,” she thought.

The laptop screen came to life, flooding the dark room with a dim artificial light. She found the file with the latest version of the manuscript, and started to LaTeX the rest of the proof. The argument was as clear as day in her head, but there was a lot of work to do. A couple of sections of the paper had to be completely reworked because, she now realized, a brand new point of view had to be employed. Perhaps the most important contribution in the entire paper would be the fact that the objects the conjecture dealt with could be realized in a totally different way. It was this connection between seemingly distinct mathematical objects that had been brewing in her mind for the last few weeks. Now it was a matter of verifying the delicate details of the relevant isomorphisms, then establishing a dictionary between objects, and then translating the problem from one category to another. Finally, some of her previous work could be used to deal with the objects on the other side of the bridge she was building.

Claudia typed frantically, without bothering to look at a clock. After so many days staring at a blank screen without progress, it was exhilarating to move forward, to write down the theorems, corollaries, and proofs that she had hoped for. Her heart raced with each proof she concluded, as she approached the end of the paper.

At some point, Rick had walked into the room, without her noticing. Claudia looked at him with a beaming smile.

“I think I got it, Richard,” she said.

He replied with a warm smile, tinged with a hint of melancholy, and gestured ‘go on’ with one hand towards the screen, encouraging her to keep doing whatever she was doing. Rick seemed to have aged years in the last few months, and Claudia felt a pang of guilt in her gut. Unfortunately, their relationship had suffered badly as of late, as she entrenched herself in her research. Claudia knew her moods had been foul at times, too many times to count, and their interactions were strained to say the least. But the conclusion of this article would change everything, and she promised herself to take some time off, go on vacation with him, and patch things up. This morning, though, she was so happy and excited about her work

that she could barely contain herself. Smiling and nodding at Rick, she turned back to her typing. He stayed in the room, sipping on his own cup of coffee, patiently watching her work.

Claudia lost track of time once more. Maybe an hour or two passed by, or was it four or five hours – she could not tell. One moment she was starving, and an indeterminate amount of time later she had an empty plate of crumbs next to her on the desk, her hunger satiated. Had Rick brought a sandwich? She was so engrossed in her work that she could not remember whether he had brought food or when she had devoured the items on the plate. It did not matter. She kept working, proving lemmas, reorganizing sections of the paper, computing nice examples to go along the results, checking details, drawing illustrative diagrams, including items in the bibliography, compiling and recompiling her LaTeX document.

Until she was done, until the paper was perfect. All the ideas fit together in an extremely harmonious way, and formed a beautiful theory and proof of the conjecture. Tears of joy started rolling down her cheeks. Rick was still there, crying in silence, enjoying her joy. Claudia got up and Rick embraced her, and both cried together, until she started bouncing up and down.

“I DID IT! I FUCKING DID IT!” she screamed, and they both laughed, shaking the tears from their faces.

“I need to post it on the arXiv,” she said to herself, and sat down in front of her laptop one more time.

“Claudia, no, hold on,” Rick said, but his voice was drowned in her own thoughts.

Her mind was racing, anticipating the reaction of other colleagues, when they saw the arXiv posting of her paper. How proud her parents would be, how happy friends would be for her. She imagined herself giving talks on her proof at the most prestigious seminars and conferences, and her heartbeat accelerated – how nerve-racking that would be. She even started thinking about the most pedagogical ways she could explain the proof to others.

“Claudia, please wait,” Rick pleaded, but it was too late.

Claudia logged into her arXiv account, and her breathing stopped. The screen displayed the list of papers that she had previously posted online and, at the top of the list, there was a link to a paper with the exact same title as the new paper she was about to upload.

“Claudia...,” Rick’s voice was coming from a place very far away.

Dumbfounded, she clicked on the link and the arXiv displayed a page, with the title of the paper, and an abstract, which read exactly like the abstract of the paper she was about to upload.

In this article, we settle a long standing conjecture...

Thinking that it must be some sort of bizarre bug in the arXiv, she clicked on the PDF link. The paper was downloaded in an instant, and appeared on her browser. There was no doubt it was the exact same paper, perhaps with very minor differences, but it was the same paper. Her paper. She was the sole author on this one too, and the paper claimed a full proof of the conjecture.

“Claudia, please stop and *look* at me,” pleaded Rick.

Choking up, she turned around.

“I... don’t understand... what...”

Rick held her hand, and continued.

“I am sorry, I was trying to tell you. You already proved it... once.”

As hard as she tried to process what was happening, nothing made sense, and Claudia started to hyperventilate. Her brain started to fire fast electric waves every which way. Increasingly upset and agitated, Claudia started shaking uncontrollably until her entire world went dark.

She woke up on her bed, with Rick by her side.

“What happened?” she said. Her mouth was dry and there was a hint of the metallic taste of blood in her saliva. Her body was weak and heavy.

Rick began explaining, the way he had done many times before. Over time, he had perfected the conversation they were about to have, and he knew when to stop for her questions, and when to pause for a few seconds to allow her to absorb the difficult news.

“The day you proved the conjecture for the first time, you had a seizure. You were rushed to the hospital, where they found a large tumor in your brain, and severe internal hemorrhaging. They had to operate right away to release the pressure building up in your cortex, to save your life, and to remove as much of the tumor as they could.”

“When? When was that?” Claudia asked, afraid of the answer, while gently touching her scalp and feeling a large scar with her fingertips. The area was tender. It felt like needles were poking the skin.

“That happened ten years ago,” Rick said, and waited for her to process what that meant.

After a long pause, Rick explained that as a side effect of the surgery, she had lost the ability to retain new memories. The damage was so severe that her brain essentially reset every night, and her mind went back to the morning of the day she had the seizure, the day she proved the conjecture.

“But... my paper... it was not finished in my laptop when I woke up... how... Wasn't it finished already?”

“I decided long ago to deep freeze your computer, in the state it was the morning of the seizure. The computer and its contents are restored to the same saved configuration every night.”

“Wh... Why?”

“Because that was the happiest day of your life... the first half of the day, obviously. The morning, the afternoon, the hours you spent finishing the paper and completing a proof of the conjecture. I've never seen you happier than that, not even close.” He paused. She felt a bit of guilt, and he knew it because they had had this conversation many times. “It's okay, Claudia. I know how important your work is to you, and I respect that.”

They looked into each other's eyes for a few seconds and then, he continued.

“Many nights, you dream of the proof, and wake up before me, jump out of bed, ready to prove it, blissfully unaware of the surgery, the coma, and the last few years. The proof itself is what your brain was trying to process when the trauma happened, so it is still probably there but fragmented. Somehow, though, you can build the proof again, and it makes you immensely happy. I... I can't bear to take that away from you.”

They embraced for a long time. When the hug ended, there was a silence, while she tried to assimilate the strangeness of the whole situation. Claudia was afraid to ask her next question, but he knew what was on her mind, so he reached for his phone, and opened the YouTube app.

“Watch this playlist,” and he handed the phone to her.

The first video was a message from her parents, assuring her they were doing well, sending their love, thanking Rick for taking such good care of their daughter and, at the end of the message, they went through a quick recap of family updates that spanned the last few years.

“We update the videos every few months,” said Richard, “and yeah, your parents are ‘youtubers’ now,” which confused Claudia, who didn’t quite recall what a youtuber was, if she had ever known what it meant.

When the second video started, a woman introduced herself as the neurologist who was treating her. The doctor explained her medical situation, the reasons for the emergency procedure, the outcome, and what she could expect in the future. In short, the brain was resilient, and there was hope that it would recover to the point that she could retain memories, but it could take many years for this to happen.

In the third video, Sarah Beischel, one of her colleagues and a good friend, appeared on the screen. She looked older than Claudia remembered her but, then again, Claudia had not looked at herself in a mirror yet, nor she was looking forward to it. After saying how sorry she was about her “condition,” Sarah narrated what happened after the paper was published to the arXiv. As Claudia had predicted, the article caused an immediate uproar. Without knowing the reason for Claudia’s silence after the paper was uploaded, her colleagues imagined her inbox was flooded with requests to speak at seminars. After the community found out she was in a coma, money was collected using a GoFundMe to help Richard with the medical expenses, quickly reaching and far surpassing the initial goal. In the meantime, Sarah had submitted the paper to the *Annals of Mathematics* on Claudia’s behalf, and it was quickly refereed by a team of experts. The article was accepted in record time, with a few very minor corrections, which Sarah took care of. Thus, the conjecture was indeed settled, and it was now called Suárez’s Theorem. The collection of tools that Claudia had invented in the paper were now commonly referred to as Suárez Cohomology. Sarah and a few other colleagues organized a five-day conference in Claudia’s honor, to go over the proof of the conjecture and do a deep dive into Suárez’ cohomological theory and its consequences. In the video, Sarah displayed the poster for the conference, which included a rather impressive list of speakers, with all the biggest names in the field. Claudia shuddered at the thought of all those mathematicians knowing who she was and discussing her work. Sarah, excitedly, continued her monologue, letting Claudia know that her article and her theory had had all sorts of follow-up papers with very interesting applications, so some of the talks in the conference were announcements of other results that stemmed from her work.

“The best part though?” Sarah asked rhetorically and continued “the best part is that you were part of the conference.”

Claudia woke up from her coma a few days before the conference started, and on the last day of the event, they were able to connect briefly via video-conference with Claudia at the hospital. On that day, Richard explained to her what had happened, and Claudia got ready to address the audience of the conference, for a few minutes.

“It was the most emotional conference I’ve ever been to,” said Sarah, tearing up a bit. “After admiring your work and its consequences for five days, we were all bawling our eyes out when you came up on the screen.”

Sarah’s video ended, and a new video began. It was an official video from the International Mathematical Union. As soon as Claudia saw the International Congress of Mathematicians acronym (ICM) on the screen, a chill traveled down her spine. Her eyes were wide open and

she turned to look at Richard, with a *'is this what I think it is??'* expression. Richard paused the video, went to her desk, and picked up a small blue velvet box from one of the drawers. Claudia was trembling when Rick gave her the box, and he worried she was about to have another seizure. She was overwhelmed and shaking but, thankfully, it was not a full-blown event. The box contained a medal, the Fields Medal. Rick gave her another long hug to calm her down.

"You are an amazing woman."

The video resumed, and she watched the ceremony, holding the large medallion in her hands. Claudia was in a wheelchair and Rick helped her onto the stage, where she received the medal. She looked disoriented on the screen, but, with a radiant smile, as she knew full well the honor that was being bestowed upon her.

The last video began playing, and three young women appeared on the screen. She did not know any of them, and she was immediately concerned that her memory was failing her. As soon as the video started, one of the women assured Claudia that she did not know them, but her work had had a huge impact on their lives. They were three Ph.D. candidates, from three top institutions (two in the USA, one in Europe), whose dissertations were intimately related to Claudia's work. They just wanted to thank her for being such a brilliant role model for many women in mathematics.

When the video playlist ended, Claudia was overwhelmed with a host of emotions.

"Do we go through this every day? Is every day like this?" she asked sheepishly.

"Some days are better than others," Rick replied honestly. "The days when you prove the theorem once again are the best, because you are the happiest you can be. Some days, though, your head hurts so much that you cannot get out of bed. Traveling is also hard on you, because you are extremely confused when you wake up elsewhere. Still, we have gone places, like the trip to the ICM in Montreal."

Claudia had many more questions but her head was pounding. They lay down in bed to rest, and she fell asleep in his arms.

When she regained consciousness, it was dark outside. Her mind was foggy, but she was aware that they had had an intense day. She actually remembered bits and pieces of the videos they had watched, she knew about a condition, but she could not recall what exactly had happened in the morning.

Rick had prepared a simple dinner, mostly leftovers, and they ate together while Claudia asked him a few more questions about their life, such as it was.

After dinner, Claudia was exhausted. She hugged Rick for a long time, gave him a kiss, thanked him "well, for everything," though she was unsure what the scope was, and went to bed. A headache kept her awake, and she tossed and turned, until she fell into a light slumber. She woke up, sweating, had a sip of water, and then tried to sleep again. This time, she slowly faded into a deeper form of unconsciousness, while math concepts started to creep into her thoughts.

In the spare room, her laptop turned to life on its own, and started running a process to revert any changes to files that had been made the previous day. At the same time, in Claudia's dreams, a surgeon was trying to save her life by removing a cohomological theory from her

brain. With the first incision, she woke up, now sweating profusely, still feeling the pain of sharp needles digging into her scalp. Groggy and confused after the disturbing nightmare, she had a few sips of water in an effort to clear her head. When she tried to go back to sleep, the tossing and turning resumed, while thoughts raced through her head, running over all the possible ways in which she could put together the last pieces of her mathematical puzzle, to prove the conjecture.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CONNECTICUT, STORRS, CT 06269, USA

E-mail address: `alvaro.lozano-robledo@uconn.edu`

URL: <http://alozano.clas.uconn.edu/>